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A CHARACTERISTIC SUBGROUP OF $\mathcal{N}$-STABLE GROUPS

BY

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ABSTRACT

Let $\bar{\mathcal{N}}$ be the class of quasinilpotent groups. Let K be an $\bar{\mathcal{N}}$-injector of a group G. In this note we study the normality in G of the subgroup $ZJ(K)$. Using this subgroup we obtain a "factorization of Thompson type" of G.

Introduction. Notation

In this note, all groups will be finite. $\mathcal{N}$ will denote the class of nilpotent groups. The concept of semisimple groups is taken from Gorenstein–Walter's paper ([7]). In particular, $L(G)$ denotes the semisimple radical of the group G.

A group G is $\mathcal{N}$-constrained if $C_G(F(G)) \leqq F(G)$ ([10]), and it is equivalent to $L(G)=1$ ([11]). $C_G(L(G))$ is the $\mathcal{N}$-constrained radical of the group G ([9]).

For every Fitting class $\mathcal{F}$ of finite groups, $\mathrm{Inj}_{\mathcal{F}}(G)$ denotes the set of all $\mathcal{F}$-injectors of the group G, that is, the set of all $H \leqq G$ such that for each $N \trianglelefteq\trianglelefteq G$, $H \cap N$ is an $\mathcal{F}$-maximal subgroup of N.

The remainder of the notation is standard and it is taken mainly from ([6]). In particular, $\pi(G)$ is the set of all primes which divide the order of the group G. $[B, A, A]$ denotes the triple commutator $[[B, A], A]$ of two subgroups A, B of G. We define inductively $[B, A; 0] = B$, and $[B, A; i] = [[B, A; i-1], A]$, for $i > 0$. $L_m(G)$ denotes the mth term of the lower central series of the group G, for $m \geqq 1$. Moreover, $d(G)$ is the maximum of the orders of the Abelian subgroups of G. Let $\mathcal{A}(G)$ be the set of all Abelian subgroups of order $d(G)$ in G. Then, as in ([6]), $J(G)$ is the subgroup of G generated by $\mathcal{A}(G)$, that is, the Thompson subgroup of G.

Glauberman ([6]), basing his work on some results and concepts of Thompson, obtained the following

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THEOREM. *Let G be a group with $O_p(G) \neq 1$ which is p-constrained and p-stable, p odd. If P is an S_p-subgroup of G, then*

$$G = O_{p'}(G)N_G(ZJ(P)).$$

In particular, if $O_{p'}(G) = 1$, then $ZJ(P) \lhd G$.

Using this subgroup, a Thompson factorization of the group

$$G = N_G(J(P))C_G(ZJ(P)) = N_G(J(P))C_G(Z(P))$$

is obtained whenever G is a p-constrained and p-stable group with $O_{p'}(G) = 1$, p odd.

Later, Mann ([10]) proved the following

THEOREM. *Let G be an $\mathcal{N}$-stable and $\mathcal{N}$-constrained group, with $|F(G)|$ odd. Let S be an $\mathcal{N}$-injector of G. Then $ZJ(S) \lhd G$.*

Under the same assumptions as in the above Theorem he proved:

$$G = N_G(J(S))C_G(ZJ(S)) = N_G(J(S))C_G(Z(S)).$$

Some related results were obtained by Arad in ([1]) and by Ezquerro in ([3]) by replacing the classes of p-groups and nilpotent groups by the class of π-groups (π a set of primes), and by a saturated Fitting formation, respectively.

Throughout these results we note that stability, constraint and existence of a unique conjugacy class of injectors are constant assumptions to obtain a factorization of Thompson type of a group.

Let $\tilde{\mathcal{N}}$ be the class of quasinilpotent groups, i.e., $\tilde{\mathcal{N}} = (G/G = F(G)L(G) = F^*(G))$.

Blessenohl and Laue ([2]) proved that all groups have a unique conjugacy class of $\tilde{\mathcal{N}}$-injectors, moreover, the $\tilde{\mathcal{N}}$-injectors of a group G are the maximal $\tilde{\mathcal{N}}$-subgroups of G containing $F^*(G)$ and if K is an $\tilde{\mathcal{N}}$-injector of G, $K = L(G)I$, where I is an $\mathcal{N}$-injector of the $\mathcal{N}$-constrained radical of G, $C_G(L(G))$.

On the other hand, it is well known that $C_G(F^*(G)) \leqq F^*(G)$ for every group G ([11]).

DEFINITION 1. Let $\mathcal{F}$ be a Fitting class.

A group G is said to be $\mathcal{F}$-stable if whenever A is an $\mathcal{F}$-subgroup of G and B is an $\mathcal{F}$-subgroup of $N_G(A)$ such that $[A,B,B] = 1$ then $B \leqq (N_G(A) \bmod C_G(A))_{\mathcal{F}}$.

The aim of this paper is mainly to prove the following results:

THEOREM A. *Let G be an $\mathcal{N}$-stable group, where $1 \neq F(G)$ is not a 2-group. Let K be an $\tilde{\mathcal{N}}$-injector of G. Then $1 \neq O_{2'}(ZJ(K)) \lhd G$.*

COROLLARY 1. *Under the same assumptions as in Theorem A*

$$G = N_G(J(K))C_G(O_{2'}(ZJ(K))) = N_G(J(K))C_G(O_{2'}(Z(K))).$$

COROLLARY 2. *Let G be an $\mathcal{N}$-*stable group, with $1 \neq |F(G)|$ *odd. Let K be an $\tilde{\mathcal{N}}$-injector of G. Then $1 \neq ZJ(K) \lhd G$.*

COROLLARY 3. *Under the same assumptions as in Corollary 2*

$$G = N_G(J(K))C_G(ZJ(K)) = N_G(J(K))C_G(Z(K)).$$

First, we give some preliminary results:

LEMMA 1. *Let A be a semisimple subgroup of a group G. Let B be a subgroup of $N_G(A)$. Suppose that there exists a positive integer $n \geqq 1$ such that $[A, B; n] = 1$. Then $[A, B] = 1$, i.e., $B \leqq C_G(A)$.*

PROOF. By induction on the order of A. Let n be the least positive integer such that $[A, B; n] = 1$; we can assume $n > 1$.

Using an inductive hypothesis and the three-subgroup lemma together with the perfectness of A, we can consider $Z(A) = 1$. Then A is a direct product of nonabelian simple groups.

Since $[A, B; n-1]$ is subnormal in A, $[A, B; n-1]$ is the direct product of certain components of A and $A = [A, B; n-1] \times K$, where K is a proper semisimple subgroup of A normalized by B.

By the inductive hypothesis $[K, B] = 1$, and easily $[A, B] = 1$.

LEMMA 2. *If G is an $\mathcal{N}$-stable group, then G is an $\tilde{\mathcal{N}}$-stable group.*

PROOF. Let A and B be $\tilde{\mathcal{N}}$-subgroups of G such that B normalizes A and $[A, B, B] = 1$.

Using the $\mathcal{N}$-stability of G we have $F(B) \leqq F(N_G(F(A)) \bmod C_G(F(A)))$. Then $F(B)C_G(F(A)) \trianglelefteq\trianglelefteq N_G(F(A))$.

Since $[A, B, B] = 1$, using the three-subgroup lemma together with the perfectness of $L(B)$, we have $[A, L(B)] = 1$, that is, $L(B) \leqq C_G(A) \leqq C_G(F(A))$. Thus $BC_G(F(A)) = F(B)C_G(F(A)) \trianglelefteq\trianglelefteq N_G(F(A))$.

On the other hand, since $[A, B, B] = 1$, by Lemma 1, $B \leqq C_G(L(A))$.

Thus, $BC_G(F(A)) \cap C_G(L(A)) = B(C_G(F(A)) \cap C_G(L(A))) = BC_G(A)$, and $BC_G(A) \trianglelefteq\trianglelefteq N_G(F(A)) \cap C_G(L(A)) \trianglelefteq N_G(F(A)) \cap N_G(L(A)) = N_G(A)$. That is, $BC_G(A) \trianglelefteq\trianglelefteq N_G(A)$.

Moreover, $BC_G(A)/C_G(A)$ is a group of automorphisms of A which stabilizes the series $A \trianglerighteq [A,B] \trianglerighteq [A,B,B] = 1$.

Using ([8]), $BC_G(A)/C_G(A)$ is a nilpotent group.

Thus, we obtain $B \leqq F(N_G(A) \bmod C_G(A))$ and trivially $B \leqq F^*(N_G(A) \bmod C_G(A))$.

LEMMA 3. *Let G be an $\mathcal{N}$-stable group. Let K be an $\tilde{\mathcal{N}}$-injector of G. Then $ZJ(K) \leqq F(G)$.*

PROOF. Using Lemma 2 and its proof on $F^*(G)$ and $ZJ(K)$ we obtain $ZJ(K) \leqq F(G \bmod C_G(F^*(G)))$.

Hence $ZJ(K)C_G(F^*(G)) \trianglelefteq\trianglelefteq G$.

Since $C_G(F^*(G)) \leqq F^*(G)$, $ZJ(K) \trianglelefteq\trianglelefteq G$. Then, $ZJ(K) \leqq F(G)$.

PROOF OF THEOREM A. (This proof is based, in part, on Glauberman's proof of his ZJ-Theorem ([6, Th. 8, 2.10]).)

We know that $K = L(G)I$, where I is an $\mathcal{N}$-injector of the $\mathcal{N}$-constrained radical of G, $C_G(L(G))$, and we easily get $I = F(K)$.

As a consequence of this, we easily obtain that the following are equivalent: (i) $F(G) \neq 1$; (ii) $F(K) \neq 1$; (iii) $ZJ(K) \neq 1$.

Moreover, if $ZJ(K)$ is a 2-group then $F(G)$ is a 2-group.

Note, too, that $O_{2'}(ZJ(K))$ normal in G implies $O_{2'}(ZJ(K))$ characteristic in G.

By Lemma 3, $ZJ(K) \leqq F(G)$. Now, to obtain the Theorem it is enough to prove that if B is a normal, nilpotent subgroup of G, then $O_{2'}(ZJ(K)) \cap B$ is normal in G.

Assume that the result is false and suppose that G is a group of least order for which the result is false. Suppose that B is a normal, nilpotent subgroup of G of least order such that $O_{2'}(ZJ(K)) \cap B$ is not normal in G.

Set $Z = ZJ(K)$ and let B_1 be the normal closure of $O_{2'}(Z) \cap B$ in G, then $O_{2'}(Z) \cap B_1 = O_{2'}(Z) \cap B$. Hence, by our minimal choice of B, we must have $B = B_1$.

Furthermore, $O_{2'}(Z) \cap B'$ is a normal subgroup of G because $B' < B$.

Since $O_{2'}(Z) \trianglelefteq K$, for any x in G we have

$$[(O_{2'}(Z) \cap B)^x, B] = [O_{2'}(Z) \cap B, B]^x \leqq (O_{2'}(Z) \cap B')^x = O_{2'}(Z) \cap B'.$$

Since B is generated by all such $(O_{2'}(Z) \cap B)^x$, it follows that $B' \leqq O_{2'}(Z) \cap B'$. Thus

$$B' \leqq O_{2'}(Z). \tag{1}$$

In particular, $O_{2'}(Z)\cap B$ centralizes B'. Since $B'\trianglelefteq G$, we have that B centralizes B', whence $B'\leqq Z(B)$. Therefore

$$\text{(2)} \qquad \mathrm{cl}(B)\leqq 2.$$

Let $A\in\mathscr{A}(K)$; we know that $K=L(G)I$, where I is an $\mathcal{N}$-injector of $C_G(L(G))$.

For every $a\in A$, we define $a_I=\{f\in I\mid af^{-1}\in L(G)\}$. Let $F=\bigcup_{a\in A}a_I\leqq I$.

Now, since $F(G)$ centralizes $L(G)$, for every positive integer m, $[B,A;m]\leqq[B,F;m]\leqq L_{m+1}(I)$. Thus, it is clear that for some positive integer n,

$$\text{(3)} \qquad [B,A;n]=1.$$

Since (1), (2), (3), $B\lhd G$ and $[A,B]'$ is of odd order, we are in the hypothesis of ([1, 2.8]); then there exists an element A in $\mathscr{A}(K)$ such that $B\leqq N_G(A)$, hence $[B,A,A]=1$.

Since G is $\mathcal{N}$-stable, $A\leqq F(G \mod C)=T$, where $C=C_G(B)\leqq C_G(O_{2'}(Z)\cap B))$.

Hence, $A\in\mathscr{A}(K\cap T)$ and, then, $J(K\cap T)\leqq J(K)$.

If $T=G$, then G/C is a nilpotent group, so KC is a subnormal subgroup of G.

Now, $KC<G$ by our choice of G and B and let M be a normal proper subgroup of G such that $K\leqq M$.

By our minimal choice of G, we have $O_{2'}(Z)\trianglelefteq M$, and then $O_{2'}(Z)$ char M. Therefore, $O_{2'}(Z)\trianglelefteq G$, contrary to the fact that $B\cap O_{2'}(Z)$ is not a normal subgroup of G. Therefore $T\lhd G$.

Since $J(K\cap T)\leqq J(K)$, then $ZJ(K)\leqq ZJ(K\cap T)$ and $O_{2'}(Z)\cap B\leqq O_{2'}(Z)\leqq O_{2'}(ZJ(K\cap T))$.

By the minimal choice of G, $O_{2'}(ZJ(K\cap T))\trianglelefteq T$. Thus, we have $O_{2'}(ZJ(K\cap T))\trianglelefteq G$. Then $B\leqq O_{2'}(ZJ(K\cap T))$.

Therefore, we conclude that B is abelian.

If $J(K)\leqq J(K\cap T)$, then $J(K)=J(K\cap T)$, so $O_{2'}(Z)=O_{2'}(ZJ(K\cap T))\trianglelefteq G$, contrary to our choice of G.

Thus, there exists an element A_1 in $\mathscr{A}(K)$ such that $A_1\not\leqq T$. Then, we must have $[B,A_1,A_1]\neq 1$.

Among all such choices of A_1, choose A_1 so that $|A_1\cap B|$ is maximal.

If B normalizes A_1, $[B,A_1,A_1]=1$, contrary to our choice of A_1. Hence, by ([1, 2.5]), there exists an element A^* in $\mathscr{A}(K)$ such that $A_1\cap B<A^*\cap B$ and A^* normalizes A_1.

Because of the maximal choice of A_1, it follows that $A^*\leqq K\cap T$. Hence $ZJ(K\cap T)\leqq A^*$. But $B\leqq O_{2'}(ZJ(K\cap T))$. Therefore,

$$[B, A_1, A_1] \leqq [O_{2'}(ZJ(K \cap T)), A_1, A_1] \leqq [A^*, A_1, A_1] = 1$$

and this is contrary to the fact that $[B, A_1, A_1] \neq 1$.

PROOF OF COROLLARY 1. Let $Z = ZJ(K)$, $C = C_G(O_{2'}(Z))$.

Since $O_{2'}(Z) \lhd G$, also $C \trianglelefteq G$. Hence, by the Frattini Argument, $G = CN_G(K \cap C) = CN_G(J(K \cap C))$. But, since $J(K) \leqq K \cap C$, it follows that $J(K) = J(K \cap C)$.

Moreover, since $Z(K) \leqq ZJ(K)$, we have $C \leqq C_G(O_{2'}(Z(K)))$.

Therefore, we conclude that

$$G = C_G(O_{2'}(ZJ(K)))N_G(J(K)) = C_G(O_{2'}(Z(K)))N_G(J(K)).$$

PROOF OF COROLLARY 2. Because of Lemma 3, $ZJ(K) \leqq F(G)$. Now, the result is a direct consequence of Theorem A.

PROOF OF COROLLARY 3. It is obtained from Corollary 2 following an argument similar to that of Corollary 1.

We know that in an $\mathcal{N}$-constrained group the $\mathcal{N}$-injectors are the $\tilde{\mathcal{N}}$-injectors. So, Corollaries 2 and 3 are a generalization of Mann's Theorem for not necessarily $\mathcal{N}$-constrained groups.

REMARK 1. The converse of Lemma 2 is not true in general. It is enough to take $G = \mathrm{SA}(2,5) \cong [C_5 \times C_5]\ \mathrm{SL}(2,5)$, which is $\tilde{\mathcal{N}}$-stable but is not $\mathcal{N}$-stable.

Before proving the Remark we give the following results:

LEMMA 4. *Let $\mathcal{F}$ be a Fitting class. Let A be a group of automorphisms of an $\mathcal{F}$-group G. If A stabilizes a series of G, then A is an $\mathcal{F}$-group.*

PROOF. Because of ([8]), A is nilpotent, so it is enough to prove that $C_p \in \mathcal{F}$, for all primes $p \in \pi(A)$.

Using ([12]), if $p \in \pi(A)$, then $p \in \pi([G, A])$. Moreover, $[G, A] \leqq F(G) \trianglelefteq G \in \mathcal{F}$. Hence C_p is isomorphic to a subnormal subgroup of the $\mathcal{F}$-group $[G, A]$. So $C_p \in \mathcal{F}$.

LEMMA 5. *Let $\mathcal{F}$ be a Fitting class. For a group G, the following are equivalent:*

(i) *G is $\mathcal{F}$-stable:*

(ii) *if A is an $\mathcal{F}$-subgroup of G, and x is a p-element of $N_G(A)$, for every prime $p \in \mathrm{char}(\mathcal{F}) = \{p \mid C_p \in \mathcal{F}\}$, such that $[A, x, x] = 1$, then $x \in (N_G(A) \bmod C_G(A))_{\mathcal{F}}$;*

(iii) *if* A *is an* $\mathcal{F}$*-subgroup of* G, *and* x *is an element of* $N_G(A)$ *such that* $[A,x,x]=1$, *then* $x \in (N_G(A) \bmod C_G(A))_{\mathcal{F}}$.

PROOF. (i)$\Rightarrow$(ii). Let A and x be as in (ii). Let $B=\langle x\rangle$. Since $\langle x\rangle \in \mathcal{F}$, using (i), the conclusion follows easily.

(ii)$\Rightarrow$(iii). Let A and x be as in (iii).

Let $\pi(\langle x\rangle)=\{p_1,\dots,p_n\}$. Then $\langle x\rangle=\langle x_1\rangle\times\cdots\times\langle x_n\rangle$, where $\langle x_i\rangle$ is a p_i-group, for each $i=1,\dots,n$.

Obviously, each x_i, $i=1,\dots,n$, normalizes A and satisfies $[A,\langle x_i\rangle,\langle x_i\rangle]=1$.

By Lemma 4, $\langle x_i\rangle C_G(A)/C_G(A)$ is an $\mathcal{F}$-group, for each $i=1,\dots,n$.

If p_i is such that C_{p_i} is not an $\mathcal{F}$-group, then $x_i \in C_G(A)$. Otherwise, using (ii), we have $x_i \in (N_G(A) \bmod C_G(A))_{\mathcal{F}}$. So $x \in (N_G(A) \bmod C_G(A))_{\mathcal{F}}$.

(iii)$\Rightarrow$(i). Let A and B be as in Definition 1. For every $x\in B$, A and x satisfy the assumptions of (iii). Then the conclusion is clear.

PROOF OF REMARK 1. We see a sketch of the proof.

Let $G=\mathrm{SA}(2,5)\cong[C_5\times C_5]\,\mathrm{SL}(2,5)$.

To prove that G is not an $\mathcal{N}$-stable group it is enough to take the subgroup $N=C_5\times C_5$, which is nilpotent and satisfies $C_G(N)=N$.

For every element x of $\mathrm{SL}(2,5)$ of order 5, we have $xN\neq 1$ and $[N,x,x]=1$.

If G were an $\mathcal{N}$-stable group, it would follow that $xN\in F(N_G(N)/C_G(N))=F(G/N)\cong F(\mathrm{SL}(2,5))$ and hence that $xN\in O_5(G/N)\cong O_5(\mathrm{SL}(2,5))=1$.

Let us see that G is an $\tilde{\mathcal{N}}$-stable group.

First, we note that G cannot have an $\tilde{\mathcal{N}}$-subgroup of order $5^\alpha\cdot 2^\beta$, $\alpha\in\{2,3\}$ and $\beta\in\{1,2,3\}$, $5\cdot 2^3$, $5\cdot 2\cdot 3$ or $5^\alpha\cdot 3$, $\alpha\in\{2,3\}$.

If there exists an $\tilde{\mathcal{N}}$-subgroup H of G of order $5^\alpha\cdot 2^\beta\cdot 3$, $\alpha\in\{2,3\}$ and $\beta\in\{1,2,3\}$, or $5\cdot 2^\beta\cdot 3$, $\beta\in\{2,3\}$, H is semisimple. In this case, if $x\in N_G(H)$ and $[H,x,x]=1$ we have, by Lemma 1, $x\in C_G(H)$.

If there exists an $\tilde{\mathcal{N}}$-subgroup H of G of order 2, 5, 3, $5\cdot 3$, $5\cdot 2$, $2\cdot 3$ or $5\cdot 2^2$, and $x\in N_G(H)$ with $[H,x,x]=1$, then $x\in C_G(H)$.

If H is an $\tilde{\mathcal{N}}$-subgroup of order 5^3, 2^3 or $2^3\cdot 3$, and $x\in N_G(H)$ is such that $[H,x,x]=1$, then

$$x\in H \bmod C_G(H)\leqq F^*(N_G(H) \bmod C_G(H)).$$

Let H be a subgroup of order 5^2.

If $H=N$, then $F^*(N_G(H)/C_G(H))=N_G(H)/C_G(H)$. Thus, if $x\in N_G(H)$ then $x\in F^*(N_G(H) \bmod C_G(H))$.

Assume that $H\neq N$.

If x is a p-element, $p \neq 5$, such that $x \in N_G(H)$ and $[H, x, x] = 1$, then $x \in C_G(H)$.

If x is a 5-element, $x \in N_G(H)$ and $x \notin H$ then $\langle H, x \rangle = P_5$, where P_5 is an S_5-subgroup of G.

But $P_5 \trianglelefteq N_G(H)$, and this case is concluded.

If H is an $\tilde{\mathcal{N}}$-subgroup of order 2^2 or $2^2 \cdot 3$ and $x \in N_G(H)$ is such that $[H, x, x] = 1$, then $x \in C_G(H)$, unless perhaps x has order 4 and $H_2 \neq \langle x \rangle$, where H_2 is an S_2-subgroup of H.

In this case $\langle H_2, x \rangle = P_2$ is an S_2-subgroup of G, and $x \in P_2 \trianglelefteq N_G(H)$.

REMARK 2. In general, it is not possible to obtain Theorem A and its Corollaries under the weaker assumption of $\tilde{\mathcal{N}}$-stability. The group in Remark 1 is an example of this.

PROOF. By Remark 1, $G = \mathrm{SA}(2,5) \cong [C_5 \times C_5]\ \mathrm{SL}(2,5)$ is $\tilde{\mathcal{N}}$-stable but not $\mathcal{N}$-stable.

Let $N = C_5 \times C_5$. Since $F(G) = N$ and $C_G(N) = N$, G is an $\mathcal{N}$-constrained group. Hence its $\tilde{\mathcal{N}}$-injectors are its $\mathcal{N}$-injectors, and they are its S_5-subgroups.

If K is an $\tilde{\mathcal{N}}$-injector of G, $d(K) = 25$ and $1 \neq Z(K) = ZJ(K)$ is subnormal but not normal in G.

Recently, Förster ([4]) has obtained the following theorem:

(1.α) *For every group G and every $T/Z(L(G)) \in \mathrm{Syl}_2(L(G)/Z(L(G)))$, then*

$$\varnothing \neq \mathrm{Inj}_{\tilde{\mathcal{N}}}(N_G(T)) \subseteq \mathrm{Inj}_{\mathcal{N}}(G).$$

Using this result we obtain another generalization of Mann's Theorem for not necessarily $\mathcal{N}$-constrained groups.

COROLLARY 4. *Let G be an $\mathcal{N}$-stable group where $1 \neq F(G)$ is not a 2-group. Let N be an $\mathcal{N}$-injector as in* (1.α). *Then*

$$G = L(G)N_G(J(N))C_G(O_{2'}(ZJ(N))) = L(G)N_G(J(N))C_G(O_{2'}(Z(N))).$$

PROOF. By the Frattini argument, $G = L(G)N_G(T)$. Note that $F(G) \leqq N_G(T)$.

The result is a direct consequence of Theorem A.

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